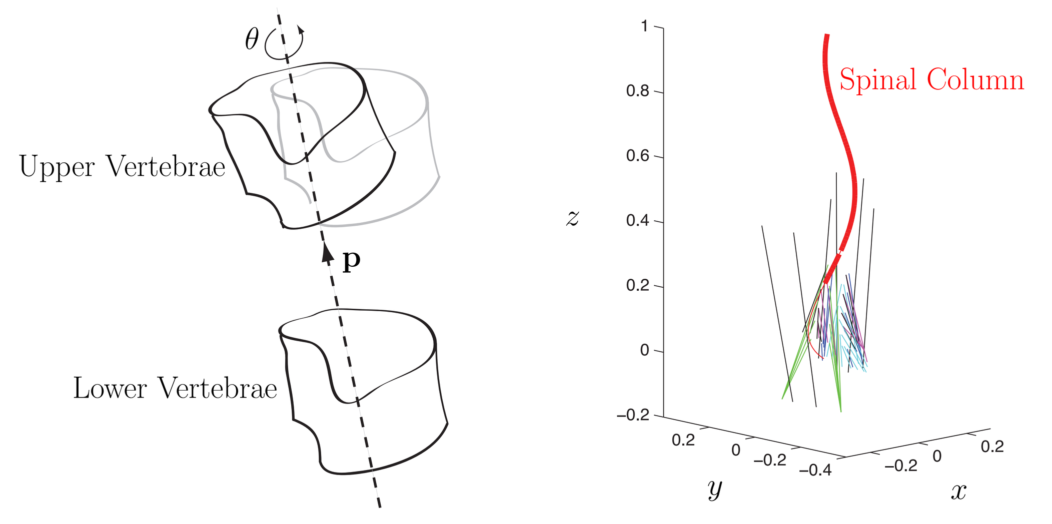
On the Dynamics of the Human Spine: Towards Mechanical Characterizations of Back Pain and its Treatments

Miguel Christophy, Nur A. Faruk Senan, David A. Moody, Oliver M. O'Reilly, and Daniel M. Peters
Department of Mechanical Engineering, University of California at Berkeley, USA
Jeffrey C. Lotz and Melodie F. Metzger
Department of Orthopaedic Surgery, University of California at San Francisco, USA

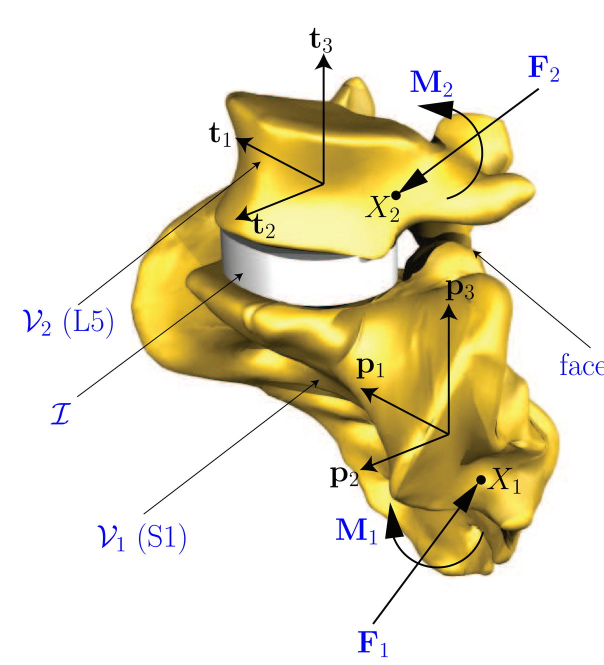
Aims of Research

The aims of the research are to characterize changes in the dynamics of the spine due to back pain therapies and total disc replacements, as well as to quantify the effects of passive and active stiffnesses. The research program features the analysis of a variety of models, experiments, and clinical data.



Intervertebral Motion

Among the components of the spine are a series of functional spinal units (FSU), each of which has 6 degrees of freedom: three translational $\mathbf{d}$ and three rotational. Rotation tensors $\mathbf{R}$ are traditionally parameterized by a 3-2-1 set of Euler angles (axial rotation, lateral bending & flexion/extension) [1]. Other components of the spine include muscles, tendons, and facet joints.



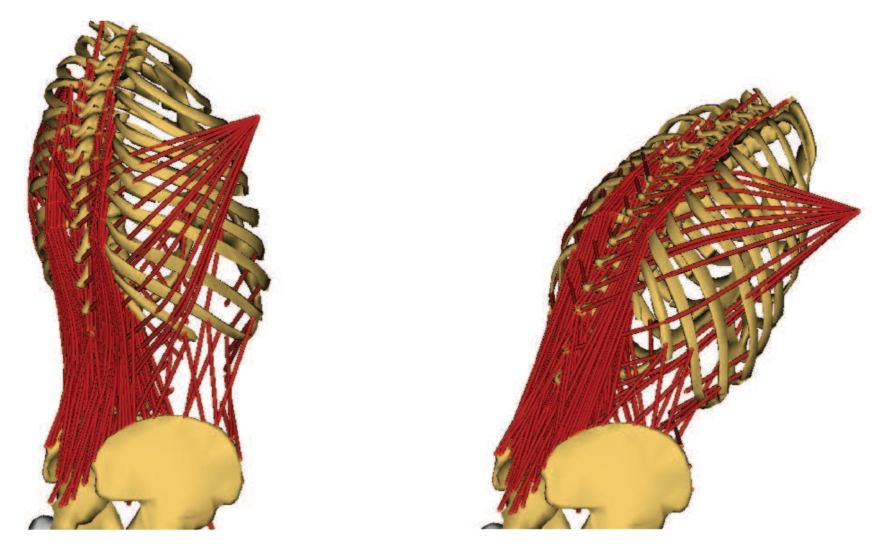
For each FSU, conservative components of the force $\mathbf{F}$ and moment $\mathbf{M}$ can be represented by the gradient of a (scalar) potential function $\mathsf{U}\left(\mathbf{d} = d_k\mathbf{E}_k, \mathbf{R}\right)$ [2]:

$$\mathbf{F}_{\text{con}} = -\frac{\partial \mathsf{U}}{\partial d_k}\mathbf{E}_k, \qquad \mathbf{M}_{\text{con}} = -\frac{\partial \mathsf{U}}{\partial \nu^k}\mathbf{g}^k = -\frac{1}{2}\left(\mathbf{q}\frac{\partial U}{\partial q_0} - q_0\frac{\partial U}{\partial \mathbf{q}} + \mathbf{q} \times \frac{\partial U}{\partial \mathbf{q}}\right).$$

Performing a Taylor expansion of $\mathbf{F}_{\text{con}}$ and $\mathbf{M}_{\text{con}}$ produces a variety of representations for Cartesian and non-Cartesian stiffness matrices [3, 4]. These concepts were applied to the spine and used to analyze total disc replacements in [4].

Multibody Models of the Spine

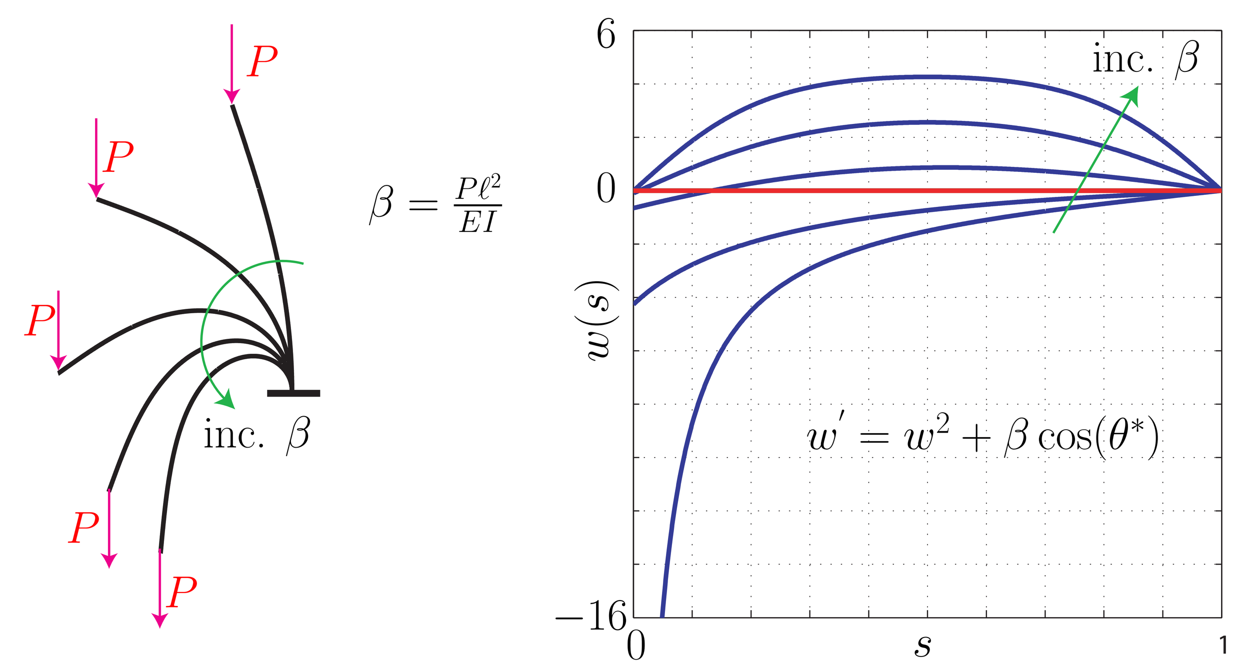
Using the OpenSim software program, a comprehensive multibody model of the lumbar spine was developed featuring 238 muscle fascicles. The model, which is discussed in [5], incorporates Thelen-type models for the muscle forces, and is based on an extensive literature survey comprising over 50 references with data on muscle forces and anatomy.



Currently, we are working on incorporating stiffness matrices for the intervertebral joints and developing a simpler model to use with a spine testing facility. Although recent data has been used to develop the musculoskeletal model, significant validation remains to be performed.

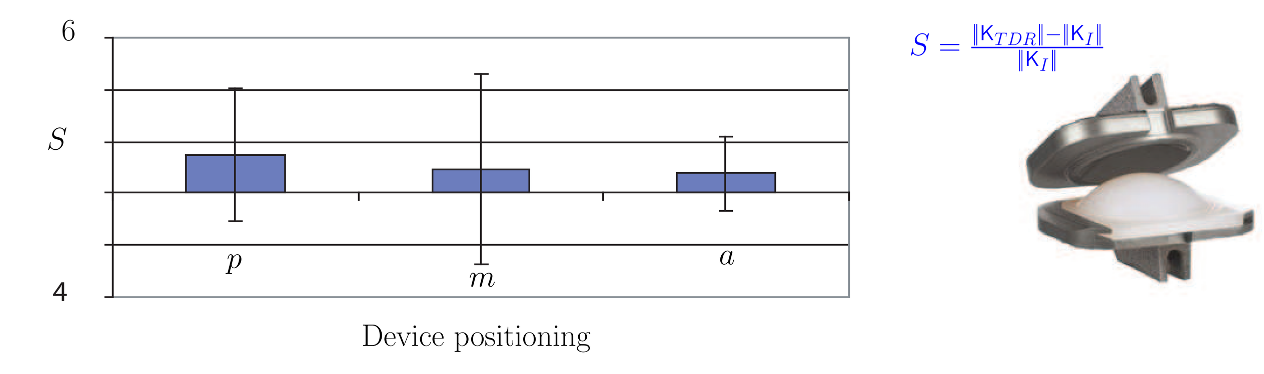
Buckling and Stability

We are developing models based on rod theory to examine spine stabilization in the presence of non-conservative muscle forces [6, 7]. Based on the second variation of the total energy of the rod-based model, a method for determining the stability of rod configurations has been developed. This method involves obtaining the solution of a Riccati differential equation. If a bounded solution exists, the energy functional is minimized and the rod configuration is stable (see results in the figure below). However, if a bounded solution cannot be obtained, then the method strongly suggests instability of the configuration being analyzed.



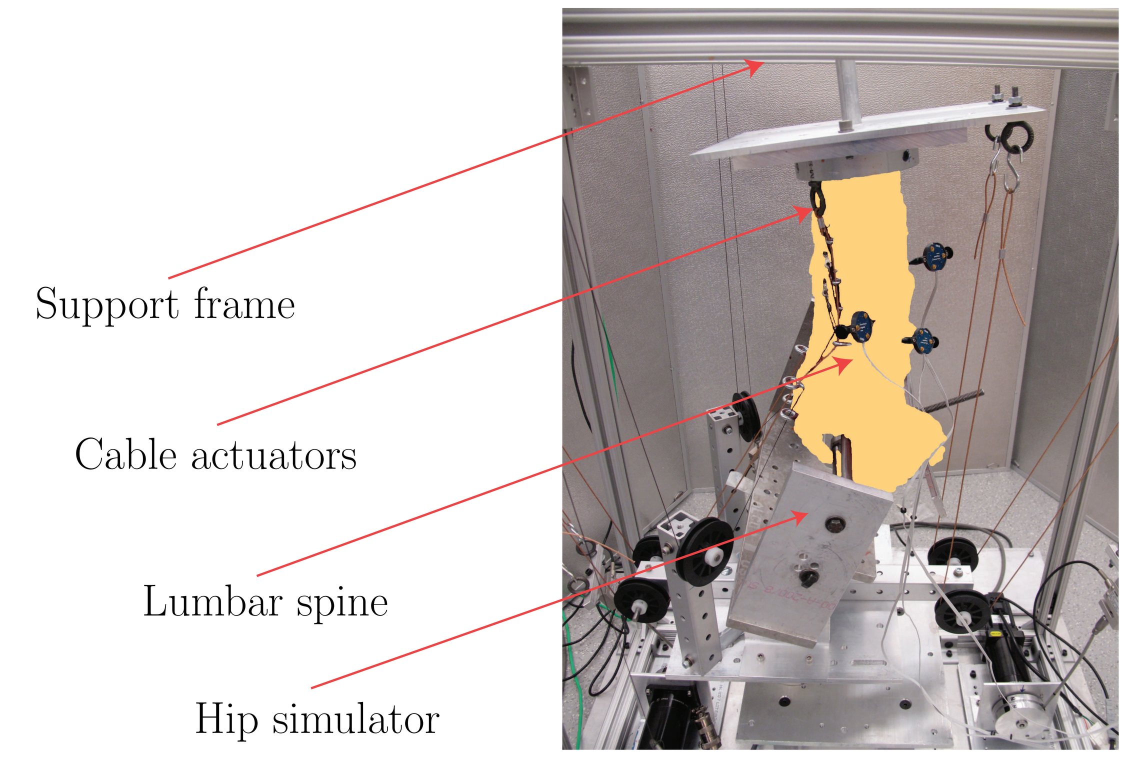
Total Disc Replacements

With the aim of characterizing the changes in stiffness due to a total disc replacement, measurements of the norm of a stiffness matrix K were determined in [4] . A sample of some of our results are shown in the figure below.



Spine Testing

In order to impose physiologic boundary conditions and muscle forces which more accurately simulate *in vivo* conditions, to quantify the role of the various muscle groups in the lower spine, and to calibrate our models, a novel spine testing facility has also been developed.



Acknowledgments and Publications

The work of the authors was partially supported by the National Science Foundation under Grant No. 0726675. Dr. Eduardo A. Misawa is the Program Director. Synthes Inc. is also gratefully acknowledged for their financial support and donation of materials. Prof. Scott Delp and his group at Stanford generously provided many hours of software support for OpenSim.

[1] Metzger, M. F., Faruk Senan, N. A., O'Reilly, O. M., and Lotz, J. C., 2010. "Minimizing errors associated with calculating the helical axis for spinal motions". *J. Biomech.,* **43**(14).

[2] O'Reilly, O. M., 2007. "The dual Euler basis: constraints, potentials, and Lagrange's equations in rigid body dynamics". *ASME J. Appl. Mech.,* **74**(2), pp. 256–258.

[3] Metzger, M. F., Faruk Senan, N. A., and O'Reilly, O. M., 2010. "On Cartesian stiffness matrices in rigid body dynamics: An energetic perspective". *Multibody Syst. Dyn.,* **44**(4), pp. 441–472.

[4] O'Reilly, O. M., Metzger, M. F., Buckley, J. M., Moody, D. A., and Lotz, J. C., 2009. "On the stiffness matrix of the intervertebral joint: Application to total disk replacement". *Journal of Biomechanical Engineering,* **131**(8), p. 081007.

[5] Christophy, M., Faruk Senan, N. A., O'Reilly, O. M., and Lotz, J. C., 2010. "A musculoskeletal model for the lumbar spine". Submitted for publication.

[6] O'Reilly, O. M., and Peters, D. M., 2010. "On stability analyses of three classical buckling problems for the elastic strut". Submitted for publication.

[7] O'Reilly, O. M., and Tresierras, T. N., 2010. "On the static equilibria of branched elastic rods". Accepted for publication in *International Journal of Engineering Science.*